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A robust approach to the calculation of paleostress fields from fault plane data: Discussion

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THE recent paper by Will & Powell (1991) shows that a least median of squares (LMS) regression is a realistic approach to paleostress estimation. Their paper has inspirational value as it causes those of us working on paleostress to rethink the direction of our work. However, I wish to raise several problems concerning their method.

CHOICE OF INPUT PARAMETERS AND RESIDUALS

The root of the weaknesses perceived in Will & Powell (1991) is their uncritical use of procedural elements taken from previous publications. In particular, they have insufficient regard to the geometrical interdependence or independence of parameters. This is seen in their choice of residuals to be minimized in the LMS procedure. Their use of the differences between field measurements and their predicted values neglects the influence of the value of one of these parameters on the accuracy of another. Specifically, as dip (p of Will & Powell 1991) decreases towards zero, the value of the dip azimuth (their d) becomes less determinate until completely indeterminate at $p = 0$. This is no problem in itself; as we lose confidence in the accuracy of measured d, so the sensitivity of the paleostress estimate to this value diminishes also. However, the pitch angle (their i) is measured from assumed strike. So an otherwise benign error in d, and hence strike, induces a like error of opposite sign in i. For fault planes of small dip, not only may the procedure of Will & Powell (1991) register a large residual of d from an essentially good field observation, but a large residual of i results even if the predicted direction of resolved shear stress coincides with the observed striation direction. Their LMS procedure will then search for a reduced residual of i by inducing misfit between the predicted and observed directions. The resulting paleostress estimate giving least median of squares of these residuals may not be the estimate which best fits the observations.

For confidence in the procedure, the magnitude of a residual must be a measure of the misfit between the prediction and the field observation. This can be achieved by matching the choice of residuals more carefully to the geometry concerned, specifying parameters of like status (rather than differing in their degree of mutual dependence) which are not liable to indeterminacy. The raw field measurements are poorly suited in this respect. The geometry of this problem is one of *angular differences in spherical orientations*, with the particular mode of mutual dependences which that implies. The optimum choice of procedure is to search *the least median of (1-cosine) of angular misfit, for three orthogonal axes* per observation. The sum of this measure over the three axes is *invariant* under change of axes.

In problems with mutually independent parameters, geometrically represented by Cartesian co-ordinates, it is the sum of squares of the co-ordinate differences which is invariant under change of axes, being the square of absolute distance between two points. This invariance of sum of squares is the prime justification for choosing to search the 'least median of *squares*', rather than least median of any other power greater than unity, when dealing with independent parameters. Those wishing to retain a formulation in *squares* of residuals for this paleostress estimation may take comfort from the fact that (1-*cosine*) equates to twice the square of the *sine* of the half-angle of misfit. However, it is *its invariance* which justifies this function, not the form in which we choose to specify it. The (1-*cosine*) form is generally the more convenient.

The optimum choice of functions to minimize is $(1 - \mathbf{n}\cdot\hat{\mathbf{n}})$, $(1 - \mathbf{s}\cdot\hat{\mathbf{s}})$ and $(1 - \mathbf{b}\cdot\hat{\mathbf{b}})$, where $\mathbf{b}$ is the unit vector perpendicular to $\mathbf{n}$ and $\mathbf{s}$, which are calculated from field observations, and '$\hat{}$' indicates the value predicted from trial of a stress tensor, $\mathbf{T}$.

THE GEOMETRICAL CONSTRAINT

The other matters of concern arise from the choice by Will & Powell (1991) of what they consider *the* constraint equation, taken from Angelier *et al.* (1982). The form in which this equation is introduced:

$$\mathbf{s}\cdot\mathbf{T}\cdot\mathbf{n} = +\sqrt{(\mathbf{T}\cdot\mathbf{n})\cdot(\mathbf{T}\cdot\mathbf{n}) - (\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n})^2}$$

betrays its origin. The right-hand side of the equation is, by Pythagoras, the length of the normal projection of the vector ($\mathbf{T}\cdot\mathbf{n}$) on the fault plane. Comparison with an independent estimate of the magnitude of the shear stress would be a good test of the estimated $\mathbf{T}$. However, no such independent estimate is available. The left-hand side of the equation is equally derivative from ($\mathbf{T}\cdot\mathbf{n}$), being the magnitude of its perpendicular projection directly onto the line of the observed shear, $\mathbf{s}$. Provided the projection directions of the two projections coincide ($\|\mathbf{n}$), the magnitudes of each side of the equation will be identical *regardless of their value*. So, despite having a form of equating magnitudes, this constraint reduces to being the orientational requirement that ($\mathbf{T}\cdot\mathbf{n}$) be coplanar with $\mathbf{n}$ and $\mathbf{s}$. Also, whereas the sign of the left side may be + or −, that of the right is always +. They will only equate if the polarity of the projection of ($\mathbf{T}\cdot\mathbf{n}$) onto $\mathbf{s}$ agrees with the observed shear sense.

The above equation is by no means the only way of specifying the requirement that ($\mathbf{T}\cdot\mathbf{n}$) be coplanar with $\mathbf{n}$ and $\mathbf{s}$. Two alternatives will be given here, followed by a reconsideration of the shear sense constraint.

Will & Powell (1991) state that the number of constraint equations being only a third of the number of measurements "precludes the use of conventional LMS", there being n equations for $3n$ measurements if n is the number of field observations. However, if one wants $3n$ equations, it is simple to enforce coplanarity by equating the vector ($\mathbf{T}\cdot\mathbf{n}$) with the sum of its normal and shear components:

$$\mathbf{T}\cdot\mathbf{n} = (\mathbf{s}\cdot\mathbf{T}\cdot\mathbf{n})\mathbf{s} + (\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n})\mathbf{n}$$

or using the star (*) product of De Paor (1990), simply:

$$\mathbf{Tn} = \mathbf{s}{*}\mathbf{Tn} + \mathbf{n}{*}\mathbf{Tn}.$$

This form is slightly simpler than the constraint used by Will & Powell (1991) and otherwise equally acceptable. As a vector equation, it can be expanded as three equations in terms of Cartesian co-ordinates, one for each of the three axial directions of the co-ordinate frame. Each of these requires that the component of ($\mathbf{T}\cdot\mathbf{n}$) along its axial direction equate to the sum of the components of the shear stress and the normal stress in that direction. So, $3n$ equations are to hand, if optimisation of the LMS procedure should so require.

Probably a better alternative is to settle for n equations which are simple and which do not purport to be a comparison of magnitudes which are, in reality, unknown. The obvious suggestion is:

$$\mathbf{b}\cdot\mathbf{T}\cdot\mathbf{n} = 0,$$

where $\mathbf{b}$ is the unit vector perpendicular to $\mathbf{n}$ and $\mathbf{s}$.

SHEAR SENSE AND SHEAR STRESS MAGNITUDE

The two alternative constraint equations suggested above take no account of *shear sense*, whereas that used by Will & Powell (1991) does. This issue is more fruitfully considered quantitatively, in terms of *the magnitude of the shear stress* acting on a fault plane, and what constitutes the most appropriate assumed lower limit (or 'threshold' value) for generating fault movement. The equations suggested above do not apply a threshold; negative and near-zero stresses are treated as equal to large positive shear stresses in their ability to generate movement of the observed sense. The equation used by Will & Powell (1991) applies a threshold value of zero; any shear stresses, however small, are considered capable of generating fault movement, provided they are positive. Neither formulation is realistic, but "Does it matter?".

A lack of limit to the acceptable range of shear stress magnitude ought to be of no practical consequence. Some falsely acceptable observations can always arise from mismeasurement of outliers, and will go undetected. Lack of constraint on shear magnitude merely adds another way in which occasional falsely acceptable observations may be produced. A major reason for favouring the LMS approach used by Will & Powell (1991) is that a least median estimate is *not* easily corrupted by a small proportion of falsely accepted observations.

Nevertheless, if no threshold is applied, a fear will remain that the number of observations of inappropriate shear stress may be sufficient to affect the paleostress estimate. So, let us consider which range of potential observations constitutes the greater threat in Will and Powell's LMS method, those giving predicted coplanar ($\mathbf{T}\cdot\mathbf{n}$) vectors of negative shear stress or those of low positive value? The σ_1 and σ_3 directions of the former would have to reside in the fields of σ_3 and σ_1, respectively, as estimated at the start of their procedure. If these fields have already been well defined, the number of these observations, acceptable but for predicted negative shear stress (wrong shear sense), must be small. Therefore, in the context of the complete procedure adopted by Will & Powell (1991), it is the other group, that is to say observations giving predicted shear stress of *correct sign but inappropriately small magnitude*, which pose a completely hidden danger to the paleostress estimation. That is why, in terms of their procedure in particular, the complexity of incorporating a restriction on shear sense but not on small positive shear stresses has no significant advantage over the simpler formulation of '$\mathbf{b}\cdot\mathbf{T}\cdot\mathbf{n} = 0$'.

FEASIBILITY OF APPLYING A THRESHOLD SHEAR STRESS

If one wishes to apply a shear stress threshold of some positive value k, rather than of zero, this can be done by substituting ($\mathbf{T}\cdot\mathbf{n} - k\mathbf{s}$) for ($\mathbf{T}\cdot\mathbf{n}$) throughout the equation used by Will & Powell (1991), to give:

$$\mathbf{s}\cdot(\mathbf{T}\cdot\mathbf{n} - k\mathbf{s}) = + \sqrt{(\mathbf{T}\cdot\mathbf{n} - k\mathbf{s})\cdot(\mathbf{T}\cdot\mathbf{n} - k\mathbf{s}) - (\mathbf{n}\cdot(\mathbf{T}\cdot\mathbf{n} - k\mathbf{s}))^2}.$$

Such a modification would appear to be the only justification for retaining this general form of constraint equation. Unfortunately, Will & Powell (1991) have chosen a reduced stress tensor with cosine functions for the diagonal elements in their reference frame. This reduction scales the other elements of **T**, and hence any appropriate value for k, by a factor which is unpredictable, being highly dependent on principal stress orientations. Changing the form of reduced stress tensor would require such a major change that modifying their procedure to include a threshold shear stress is unlikely to be worthwhile.

In the longer term, the inverse problem addressed by Will & Powell (1991) should be tackled by a *least median* procedure which assumes appropriate parameters for a linear friction law instead of an arbitrary threshold. This would permit estimation of both a reduced stress tensor and a relative stress difference, such as $(\sigma_1 - \sigma_3)/\sigma_1$. The theoretical foundation for including the frictional constraint has already been laid by Célérier (1988) but consideration of it here would take us beyond the scope of the present discussion.

REFERENCES

Angelier, J., Tarantola, A., Valette, B. & Manoussis, S. 1982. Inversion of field data in fault tectonics to obtain the regional stress—1. Single phase fault populations: a new method of computing the stress tensor. *Geophys. J. R. astr. Soc.* **69**, 607–621.

Célérier, B. 1988. How much does slip on a reactivated fault plane constrain the stress tensor? *Tectonics* **7**, 1257–1278.

De Paor, D. G. 1990. The theory of shear stress and shear strain on planes inclined to the principal directions. *J. Struct. Geol.* **12**, 923–927.

Will, T. M. & Powell, R. 1991. A robust approach to the calculation of paleostress fields from fault plane data. *J. Struct. Geol.* **13**, 813–821.